

# “Optical Guiding” Limits on Extraction Efficiencies of Single-Pass, Tapered Wiggler Amplifiers

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## Abstract

Single-pass, tapered wiggler amplifiers have an attractive feature of being able, in theory at least, of extracting a large portion of the electron beam energy into light. In circumstances where an optical FEL wiggler length is significantly longer than the Rayleigh length  $z_R$  corresponding to the electron beam radius, diffraction losses must be controlled via the phenomenon of optical guiding. Since the strength of the guiding depends upon the effective refractive index  $n$  exceeding one, and since  $(n-1)$  is inversely proportional to the optical electric field, there is a natural limiting mechanism to the on-axis field strength and thus the rate at which energy may be extracted from the electron beam. In particular, the extraction efficiency for a prebunched beam asymptotically grows linearly with  $z$  rather than quadratically. We present analytical and numerical simulation results concerning this behavior and discuss its applicability to various FEL designs including oscillator/amplifier-radiator configurations.

## 1 Introduction

For over a decade (see, e.g. [1]), single-pass, tapered wiggler free-electron laser (FEL) amplifiers have been suggested as a means to obtain much higher extraction efficiencies of electron beam power into laser light than is normally possible in simple oscillator configurations. A key aspect of the tapered wiggler amplifier is the phenomenon of “optical guiding”[3] which permits both the optical gain length and total wiggler length to be many times that of the optical Rayleigh length. The guiding is caused by the bunched electron beam having an effective refractive index  $n$  exceeding one and thus acting as an optical fiber. Experimentally, “gain guiding” was observed in the LLNL Paladin experiment[4] but the electron beam brightness was insufficient to permit meaningful tapering experiments.

While doing extensive modeling in the mid-1980’s for the (then) upcoming Paladin experiment, I (and undoubtedly others) noticed that, well into the saturated gain regime of an FEL amplifier, the optical power grew approximately linearly with  $z$  as opposed to quadratically as would be the case if one “simple-mindedly” presumed that all the emitted optical power remained confined within the electron beam cross-section. Many years later, while examining [5] the theoretical efficiency of a two stage FEL involving an oscillator followed by a single-pass “radiator”, I realized that this linear growth rate stemmed from a fundamental aspect of optical guiding - namely, since the refrac-

tive index  $n$  scales inversely as the optical electric field, if the effective longitudinal bunching remains more or less constant, the on-axis field (and thus the instantaneous energy extraction rate) is limited to a more-or-less constant value.

## 2 Theoretical Analysis

I first adapt Colson’s normalized FEL parameters to amplifier configurations and then apply them to determine the limits of optical guiding in the saturated gain regime.

### 2.1 Normalized Variables

Colson [2] introduced a set of normalized quantities for analysis of oscillator FEL’s; with minor adaptation, they also prove useful for analysis of single-pass amplifiers. The normalized, complex RMS electric field  $a$  and the normalized current density  $j$  may be defined as

$$a \equiv \frac{2a_w f_B k_w L^2}{\gamma_o^2} \frac{e \bar{E}}{mc^2} \quad (1)$$

$$j \equiv \frac{8n_e L^3 (\epsilon \pi a_w f_B)^2}{\lambda_w \gamma_o^3 mc^2} = \frac{k_w L^3}{\gamma_o^3} \frac{\omega_p^2 a_w^2 f_B^2}{c^2} \quad (2)$$

Here  $\omega_p$  is the on-axis electron plasma frequency,  $a_w$  is the normalized rms vector potential,  $\gamma_o$  the beam’s initial Lorentz factor, and  $f_B$  is the Bessel function difference coupling term for a linearly polarized undulator, and  $L$  is a scaling length to be defined below.

Letting  $\bar{z} = z/L$ , the FEL field equation in the slowly-varying envelope approximation may be rewritten as

$$\frac{\partial a}{\partial \bar{z}} = j (\langle \zeta \sin \theta \rangle + i \langle \zeta \cos \theta \rangle) + i \frac{L \nabla_{\perp}^2 a}{2k_s} \quad (3)$$

where the brackets represent averaging over the particle phases  $\theta_i$  (measured relative to that of a plane wave),  $k_s$  is the radiation wavenumber, and  $\zeta_i \equiv a_w f_B \gamma_o / a_w^0 f_B^0 \gamma_i$  where the “o” refers to the quantity’s initial value. For  $a_w \geq 2$ ,  $\zeta_i$  varies little from one for trapped particles and for simplicity I drop it in the remainder of the analysis in this section (the particle simulations described in §3 include it implicitly).

Recognizing that  $j \propto \rho^3 \equiv \omega_p^2 a_w^2 f_B^2 / 16 \gamma_o^3 k_w^2 c^2$  where  $\rho$  is the Pierce parameter, we judiciously choose  $L \equiv \lambda_w / 4\pi \rho$ , which is (approximately) the exponential growth length for the optical electric field. Note that with this particular definition of

$L$ ,  $j = 2$  exactly and different amplifiers will have different normalized wiggler lengths ( $L_w/L$ ) but identical normalized current densities. This is in contrast to oscillators where Colson sets  $L \rightarrow L_w$  and the normalized wiggler length is always one but the normalized currents vary greatly.

## 2.2 Optical Guiding and Energy Extraction in the Saturated Gain Regime

In most proposed amplifier configurations, the input laser power at  $z = 0$  will be much smaller than the “saturation” power and the power will grow exponentially until  $P \rightarrow P_{sat} \approx \rho P_{beam}$ , where, using relation (1),  $|a| = 2$ . To increase the laser power significantly beyond this level, the wiggler must be tapered to reduce the resonant energy  $\gamma_r$  with  $z$ . Tapering will work well, however, only if the diffractive losses are not extreme. Following the analysis presented in Scharlemann, Sessler, and Wurtele[3], we expect strong, refractive guiding in both the exponential and saturated gain regions if the “fiber parameter”

$$V^2 \equiv (n^2 - 1) k_s^2 r_*^2 \quad (4)$$

is of order 1 or greater where  $r_*$  is, approximately, the  $1/\epsilon$  point in a Gaussian profile electron beam or the HWHM in a parabolic profile. From relation (3), the real part of  $n$  is given by

$$\text{Re}(n) - 1 = \frac{j}{k_s L |a|} \langle \cos \psi \rangle \quad (5)$$

and, for the usual case of  $(n - 1)$  small, one finds

$$V^2 = 4 \frac{j}{|a|} \frac{k_s r_*^2}{2L} \langle \cos \psi \rangle \quad (6)$$

where  $\psi_i \equiv \theta_i + \phi$ , measures the particle longitudinal phase relative to that of the ponderomotive well. With  $j = 2$  and  $z_R \equiv k_s r_*^2/2$  and reasonable values of  $\langle \cos \psi \rangle \geq 0.5$  and  $z_R/L \geq 1$ , optical guiding is strong at the beginning of the saturated gain regime permitting  $|a|$  to grow linearly with  $z$ .

Eventually though, when  $|a|$  approaches

$$a^* \equiv 4j \langle \cos \psi \rangle \frac{z_R}{L} \quad (7)$$

$V^2$  becomes sufficiently small that optical guiding “fails”, and significant radiation begins to leak transversely beyond  $r = r_*$ . At this point,  $|a|$  stays nearly constant with  $z$  and, since the particle deceleration is directly proportional to  $|a| \langle \sin \psi \rangle$ , the total power grows linearly with  $z$ .

With  $|a|$  constant and presuming constant values of  $\langle \sin \psi \rangle$  and  $\langle \cos \psi \rangle$ , it is easy to estimate an upper bound to the energy extraction in the saturated gain regime. Denoting  $\Delta\gamma$  as the mean reduction in the beam energy,

$$\frac{d}{dz} \left( \frac{\Delta\gamma}{\gamma_o} \right) = -\frac{1}{4\pi} \frac{|a| \lambda_w}{L} \langle \sin \psi \rangle \quad (8)$$

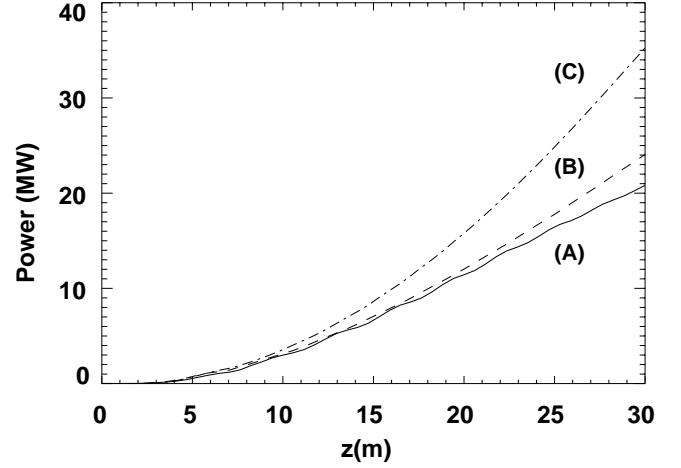


Figure 1: Total laser power versus  $z$  for FRED simulations of a  $10.6 \mu\text{m}$  FEL. Run “A” includes the full physics of diffraction and betatron motion; Run “B” has individual betatron motion suppressed; Run “C” has both betatron motion and diffraction suppressed.

Inserting the asymptotic value of  $|a|$  from eq. (7) and the definition of  $L$ ,

$$\left( \frac{\Delta\gamma}{\gamma_o} \right) = -8\rho \frac{\Delta z}{L} \frac{z_R}{L} \langle \sin \psi \rangle \langle \cos \psi \rangle \quad (9)$$

Since the maximum practical value of  $\langle \sin \psi \rangle \langle \cos \psi \rangle$  is  $\approx 0.25$ , the energy extraction per gain length  $L$  is about  $2\rho(z_R/L)$  or less.

## 3 SIMULATION RESULTS

The FRED [6] simulation code was used to investigate the phenomena described above. I chose beam parameters of  $I_B = 1 \text{ kA}$ ,  $\gamma_o = 200$ , a uniformly filled 4-ellipsoid transverse phase space with  $\varepsilon = 1400\pi \text{ mm-mrad}$  (equivalent energy spread  $\Delta\gamma/\gamma_o = 2.7 \times 10^{-3}$ ),  $\lambda_s = 10.6 \mu\text{m}$ , and a 30-m, linearly-polarized wiggler with  $\lambda_w = 80 \text{ mm}$ ,  $a_w = 3.1$ , and curved poletip focusing. The corresponding  $\rho = 7.2 \times 10^{-3}$ ,  $L = 0.89 \text{ m}$ , and  $z_R = 1.22 \text{ m}$ . With an input 10 MW laser power (*i.e.*  $|a| = 0.3$ ), 5 meters are needed for saturation with  $P_{sat} \approx 0.8 \text{ GW}$ . Beginning at 2.5 m, the wiggler  $a_w$  was tapered using a constant  $\psi_r$  strategy.

Figures 1 and 2 show results from three illustrative runs. Run A with  $\psi_r = 0.5$  included “full” physics (*e.g.* diffraction and betatron motion) and curve A in Figs. 1 and 2 show the total laser power and on-axis electric field, respectively, as functions of  $z$ . Both show behavior similar to that predicted above; namely the on-axis intensity approaches an asymptotic value and the total power grows linearly with  $z$ . Approximately 20% of the total electron beam power is converted to radiation by

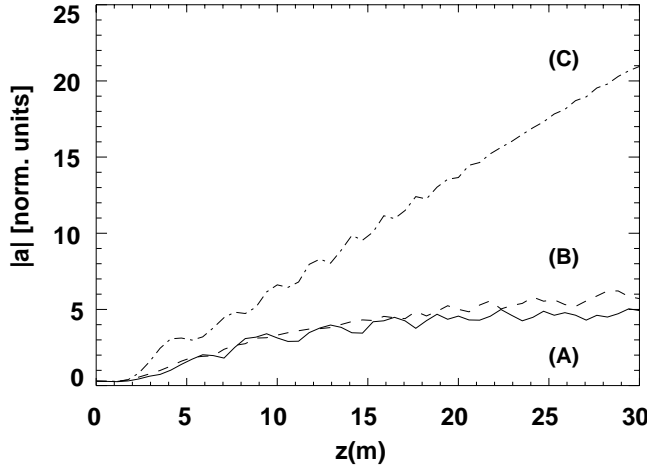


Figure 2: Normalized (see Eq. 1), on-axis radiation electric fields versus  $z$  for the three FRED runs of Fig. 1.

$z = 30$  m with bunching parameters  $b \equiv |\langle \exp i\theta \rangle|$ ,  $\langle \cos \psi \rangle$ ,  $\langle \sin \psi \rangle$  of 0.55, 0.45, and 0.30 respectively. Run B retained diffraction and beam emittance but suppressed betatron motion. As with run “A”, the on-axis field approaches a constant value but the total energy extraction increases to 24% and bunching parameters of 0.60, 0.52, and 0.31. For  $z \geq 20$  m, Fig. 2 shows that  $a \rightarrow 5.5$  in Run B; for comparison, eq. (7) predicts  $a^* = 5.7$  for  $\langle \cos \psi \rangle = 0.52$  in good agreement.

Run C suppressed both diffraction and betatron motion, thus confining the emitted light radially. (As a side note, if diffraction is suppressed but betatron motion retained, the overall amplifier performance becomes rather poor because of detrapping caused by the individual particle betatron motion through the highly curved wave fronts which arise from the radial gain variation). Diffraction appears to strongly reduce the radial phase variation and thus lessens the performance penalty induced by betatron motion. Best performance came from  $\psi_r = 0.33$  resulting in 35% energy extraction by  $z = 30$  m and  $b=0.55$ ,  $\langle \cos \psi \rangle = 0.50$ , and  $\langle \sin \psi \rangle = 0.25$ . In sharp contrast to Runs A and B which included diffraction, the on-axis electric field in Run C grows linearly with  $z$  throughout the saturated gain regime. As expected from the linear growth of  $|a|$ , the extracted power grows nearly quadratically beyond  $z \approx 8$  m.

## 4 Discussion

The dependence of the energy extraction rate [eq. (9)] in the saturated gain regime upon the product  $\langle \cos \psi \rangle \langle \sin \psi \rangle$  and hence the bunching fraction  $b$  squared emphasizes, as has long been realized, that it is crucial in the operation of an optical wavelength, single-pass, tapered wiggler FEL to both trap a large fraction of the beam in the ponderomotive well and decelerate those particles with minimal detrapping. When a waveguide is present to

confine the radiation mode as is usually true for microwave FEL amplifiers, optical guiding physics is no longer critical and the energy extraction rate should be less sensitive to bunching fraction. Although a cursory glance at eq. (9) would suggest (for a constant  $b$ ) that the extraction rate would improve if the beam radius and hence  $z_R$  increased, since  $\rho^3 \propto n_e \propto I/r_*^2$ , to lowest order this is not so and in general decreasing  $\rho$  both increases the gain length in the exponential gain regime and makes the effective energy spread due to emittance even worse. Consequently, as many experimentalists know from painful experience, it is always best to optimize beam quality, even if it means trading off a bit of peak beam current.

## References

- [1] Kroll, N.M., Morton, P.L., and Rosenbluth, M.N, “Free-Electron Lasers with Variable Parameters Wigglers”, *IEEE J. Quant. Elec.* **QE-17** (1981) 1436.
- [2] W.B. Colson, “Classical free electron laser theory”, in *Laser Handbook* (Free Electron-Lasers), eds. W.B. Colson, C. Pellegrini, and A. Renieri, (North-Holland, Amsterdam, 1990) Vol. 6, p. 115.
- [3] E.T. Scharlemann, A.M. Sessler, and J.S. Wurtele, Proc. Int. Workshop on Coherent and Collective Properties in the Interaction of Relativistic Electrons and Electromagnetic Radiation, Como, Italy, 1984, eds. R. Bonifacio, F. Casagrande, and C. Pellegrini, *Nucl. Inst. and Meth.* **A239** (1985) 29.
- [4] T.J. Orzechowski, “Free-electron lasers driven by Linear Induction Accelerators”, in *The Physics of Particle Accelerators*, eds. M. Month and M. Dienes (AIP, New York, 1992), **AIP Conf. Proc.** **249** p. 1839.
- [5] S. J. Hahn and W.M. Fawley, “Multidimensional simulation studies of the SELENE FEL oscillator/buncher followed by a radiator/amplifier output scheme”, to be published in *Proc. SPIE Photonics West*, San Jose, 1995.
- [6] R.A. Jong, W.M. Fawley, and E.T. Scharlemann, “Modeling of induction-linac based free-electron laser amplifiers” in *Modeling and simulation of laser systems*, **SPIE 1045** (1989) p 18.